PID controller design on Internet: www.PIDlab.com

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Abstract

The purpose of this article is to introduce a simple Internet tool (Java applet) for PID controller design. The applet is based on the Nyquist plot shaping method which seems to be worthwhile for industrial practitioners.

1. Introduction

PID controllers are widely used in industrial practice more than 60 years. The development went from pneumatic through analogue to digital controllers, but the control algorithm is in fact the same. The PID controller is a standard and proved solution for the most of industrial control applications. In spite of this fact, there is not some standard and generally accepted method for PID controller design based on known process model.

Over the years, there are many formulas derived to tune the PID controller. But there exist only a few universal procedures, which can be used for arbitrary order irrational or non-minimum phase transfer functions. One of these is described in [1]. The classical D-partition [2] method is used for the ideal PID controller design where the gain and phase margins are specified. For the real PID controller (filtered derivative part) and for more general design specifications, this method must be modified as it is shown in parts 2 and 3. The arising algorithm seems to be universal and still comprehensible for people in industrial practice. That is why we decided to create a Java applet accessible on Internet. The applet is shortly described in part 4. Illustrating example is given in part 5.

2. Design method principle

Consider the control loop shown in figure 1 with the PI(D) controller *C(s)* and the plant described by a stable transfer function *P(s).*

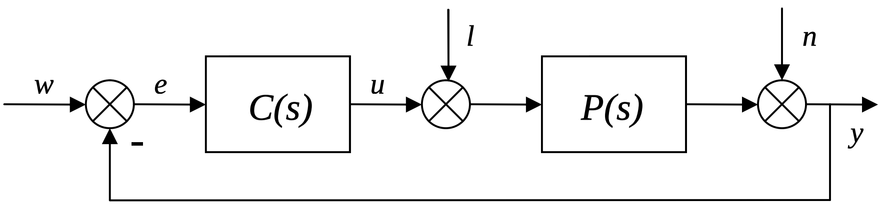


Fig. 1 Control loop

Variables *w, e* and *y* denote setpoint, control error and process variable, respectively. Variables *l* and *n* denote disturbances affecting the control loop.

It is well known, that required loop performance (e. g. gain and phase margins) could be reached by shaping of the Nyquist plot

$$L(jw) = C(jw)P(jw). \qquad (1)$$

For example, the minimum gain margin 2 (Gm>2) and minimum phase margin 60˚ ($Pm \geq 60˚$) are equivalent with requirement, that points X_1 = -1/2 and $X_2 = -1/2(1+j\sqrt{3})$ in fig. 2 should lie on the left side of $L(jw)$ or just on this curve. If $Gm < 3$ and $Pm < 90˚$ are required, the points X_3= -1/3 and X_4 = -j should lie on the right side of $L(jw)$. In such way the points X_1, X_2, X_3, X_4 define required Nyquist plot shape.

Similarly, one can proceed with, if the restriction of the sensitivity function

$$S(jw)=\frac{1}{1+L(jw)}$$

is required in a form

$$\sup_w |S(jw)| \le M_s, \tag{2}$$

or if the restriction of the complementary sensitivity function

$$T(jw)=\frac{L(jw)}{1+L(jw)}$$

is required in a form

$$\sup_w |T(jw)| \le M_p. \tag{3}$$

It can be simply proved, that condition (2) (resp. (3)) is equivalent with requirement that the Nyquist plot $L(jw)$ does not have any intersect with a circle having center

$$c = -1 \quad (\text{resp. } c=\frac{M_p^2}{1-M_p^2}) \tag{4}$$

and radius

$$R=\frac{1}{M_s} \quad (\text{resp. } R=\frac{M_p^2}{|M_p^2-1|}). \tag{5}$$

Now, consider the case, when only one shaping point $X = u+jv$ is specified in the Nyquist plot plane. Our aim is to find all possible combinations of parameters k, k_i of the PI controller with transfer function

$$C(s)=k+\frac{k_i}{s},$$

which ensure, that the point X lies on the left side of the Nyquist curve. For this purpose, let us solve an equation

$$L(jw)=(k-j\frac{k_i}{w})(a(w)+jb(w))=u+jv \tag{6}$$

for unknown k and k_i, where $a(w)=\mathrm{Re}(P(jw))$ and $b(w)=\mathrm{Im}(P(jw))$.
The relations obtained

$$k=\frac{a(w)u+b(w)v}{a^2(w)+b^2(w)}$$

$$k_i=\frac{[a(w)v-b(w)u]w}{a^2(w)+b^2(w)} \tag{7}$$

define parametric curve in the PI parameters k, k_i plane. This curve together with k, k_i axis splits the parametric plane into several regions as it is shown in fig. 2b. We are interested only in positive values of k, k_i in the first quadrant. It follows from (6) that the region border corresponds with PI parameters leading to the Nyquist plot passing through the X point. All points inside any region lead to the Nyquist plot having point X at the same side. More precisely, the number of encirclements of the point X by the Nyquist plot is the same. Usually just one region contains suitable points corresponding to the required location of the Nyquist plot and the point X.

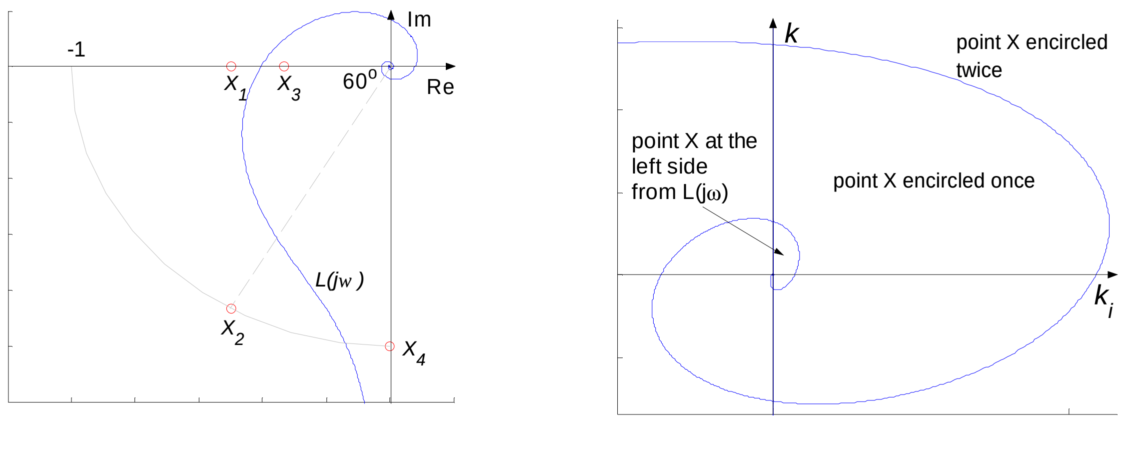
Any Nyquist plot shaping problem with a finite number of shaping points could be transferred to the one-point case described above. If we find corresponding regions R_i for all points X_i, $i=1,2,\ldots,n$, than their intersection

$$R=\bigcap_{i=1}^{n} R_i$$

contains all points solving our problem. It is suitable to choose a point with the maximum k_i coordinate from all possible solutions. The PI controller has then a maximum gain in low frequencies and also minimizes the criterion

$$IE = \int_0^{+\infty} e(t)dt \,,$$

when the step is applied on the closed loop set point.



(a) (b)

Fig. 2 (a) Nyquist plot shaping (b) Curve (7) and corresponding regions

3. PID controller design

In this part, the method presented above is used for a real 2DOF PID controller design. The controller is in according to ISA norm described by the relation

$$U(s) = k\left\{ bW(s) - Y(s) + \frac{1}{T_i s}\left[W(s) - Y(s)\right] + \frac{T_d s}{\frac{T_d}{N}s + 1}\left[cW(s) - Y(s)\right]\right\}, \tag{8}$$

where Y(s), W(s) and U(s) denote images of process variable, set point and manipulated variable. Further k denotes gain, T_i and T_d are integral and derivative time constants, b and c are weight coefficients of the set point in proportional and derivative part, and parameter N specifies the degree of derivative part filtering. It follows from relation (8), that the closed loop stability and disturbance response are dependent only on parameters k, T_i , T_d and N, while the closed loop response can be independently influenced by the parameters b and c. That is why the controller design could be divided into two steps. Firstly, we design the 1DOF PID controller (b = c = 1) described by the transfer function

$$C(s) = k\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right) \tag{9}$$

and then we tune parameters b and c manually ($b \in \langle 0,1 \rangle$ and $c=0$ are recommended). Since the controller (9) has four design parameters k, T_i , T_d and N, we relate them by additional conditions, so that the parameter plane method could be used. Parameter N has the clear physical interpretation. When $N \to \infty$, we obtain an ideal PID controller, while the derivative part is switched off if $N \to 0$.

The value of N is usually chosen in the interval $\langle 1,10 \rangle$ according to the noise in process variable. The ratio $f = T_d \,/\, T_i$ is often equal to ¼ [4]. Newer studies [3] acknowledged correctness of this value especially for plants with a monotone step response. Note, that $f = 0$ leads to PI controller and f > 1/4 enhances the derivative part. When N and f are constant, we only need to determine two parameters k and $k_i = k\,/T_i$ like in part 2. Computation of region borders is much more complicated but the design technique can be used without changes. The rule of optimal parameters choice is also the same: the optimal point has a maximum k_i coordinate. More details could be found in [5].

4. User description of the applet

Let us shortly describe the graphical user interface of the applet, which is free accessible on www.PIDlab.com
The applet area is divided into five basic windows (fig. 3)

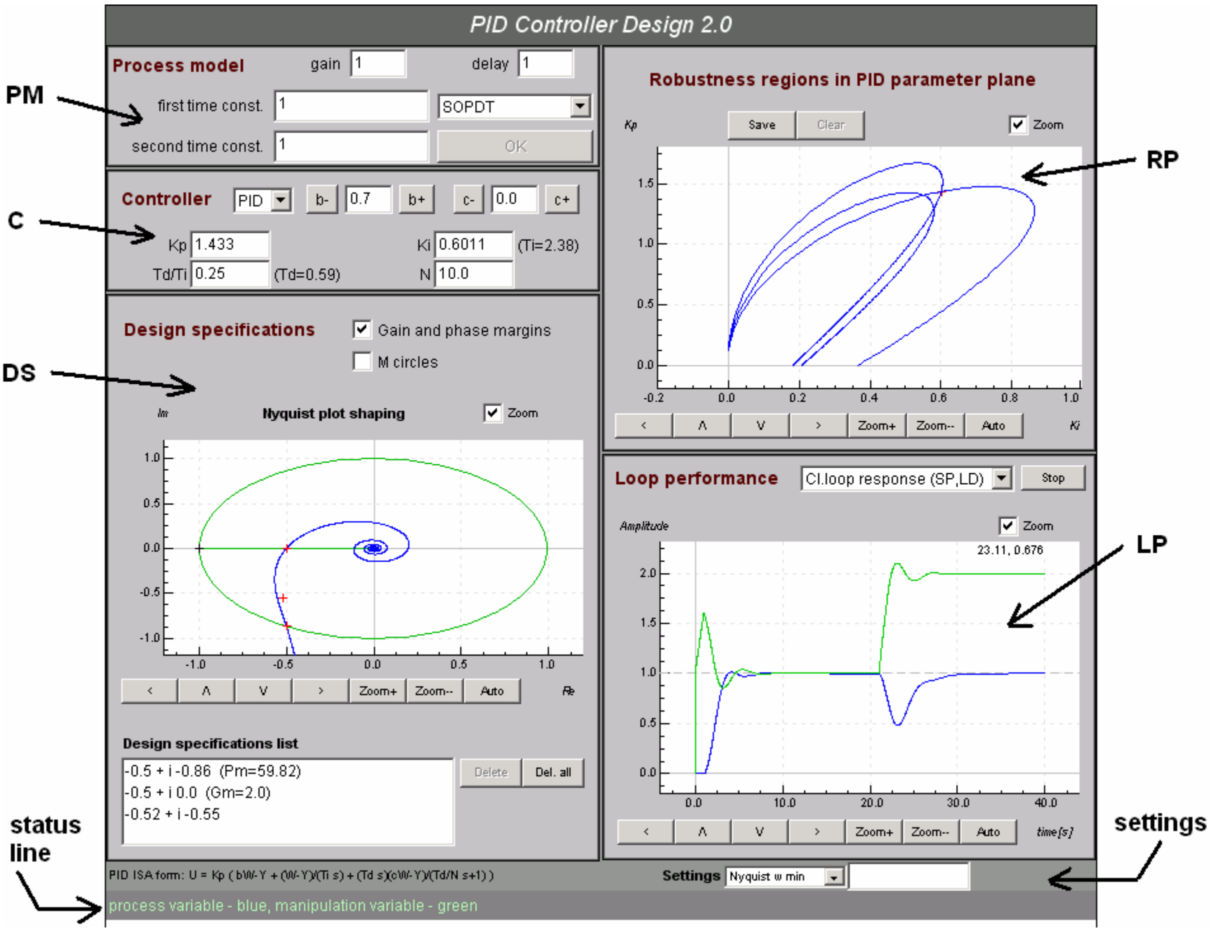


Fig. 3 General view of the applet

1) Process model (PM)

In this window, we can specify a new process model (transfer function) using one of four ways selected right in the middle. We can specify coefficients of numerator and denominator of the transfer function, time constants or zeroes and poles. The last possibility is the “second order plus dead time” form. All forms can be complemented by the gain and the transport delay. The new process model should always be confirmed by OK button.

1) Controller (C)

It is possible to set all parameters of the 2DOF ISA PID controller by hand. The controller design is divided into following steps.

1) Define new process model
2) Choose type of the controller (PI/PID). Choose the parameters *N* and *f*.
3) Define design specifications in the Nyquist plot plane (DS window).
4) By clicking on any point in satisfactory region in the RP window (the intersection of all regions corresponding to design specifications) we obtain the parameters k and k_i and related T_i, T_d.
5) Finally, we can change all parameters by hand. Especially decreasing of the parameter b could cause a lower overshot in closed loop.

3) Design specifications (DS)

In this window, we can specify general requirements on Nyquist plot shape using shaping points. We can easily specify gain and phase margins or a restriction of the (complementary) sensitivity function by special choice of these points. Points could be added by left mouse button click. All points are listed in the design specifications list. All points can be edited manually after selection in this list. Gain and phase margins can be easily specified (with the gain and phase margins checkbox checked) on the unit circle resp. on the negative real axis.

4) Robustness regions in PID parameter plane (RP)

This window is active only if at least one design specification is defined. The regions corresponding to design specifications are painted in this window. We need to click here to choose or change controller parameters k and k_i.

5) Loop performance (LP)

One of four graphs can be chosen in this window. The process step response is painted when new process is defined. Other graphs are available only if all controller parameters are specified. We can choose the closed loop set point and load disturbance step response, the sensitivity function or the complementary sensitivity function.

Graph Axes

We can change the axes ranges with buttons under each graph if zoom checkbox is checked. When we press the auto button, ranges are set automatically. The best way to zoom is to define the zoom rectangle by mouse dragging.

Settings and status line

In a lower part of the applet is the settings panel. Here we can change the frequency ranges for all characteristics. We can also set the simulation time and the period of discretization of the process and the controller (used for simulation).

The status line can be very useful. Actual applet state information as so as short help about important components are printed here.

5. Practical example

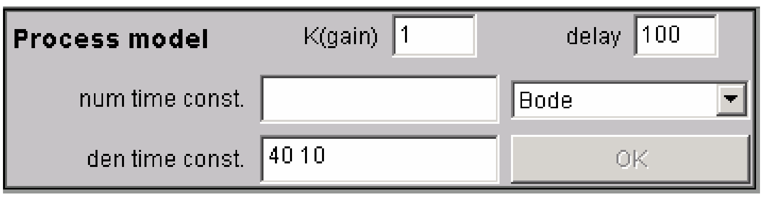
Let us use the applet for brick press controller design. The brick press is described by the transfer function

$$F(s) = \frac{e^{-100s}}{(40s+1)(10s+1)}.$$

The controller design is divided into following steps.

1) Process model definition

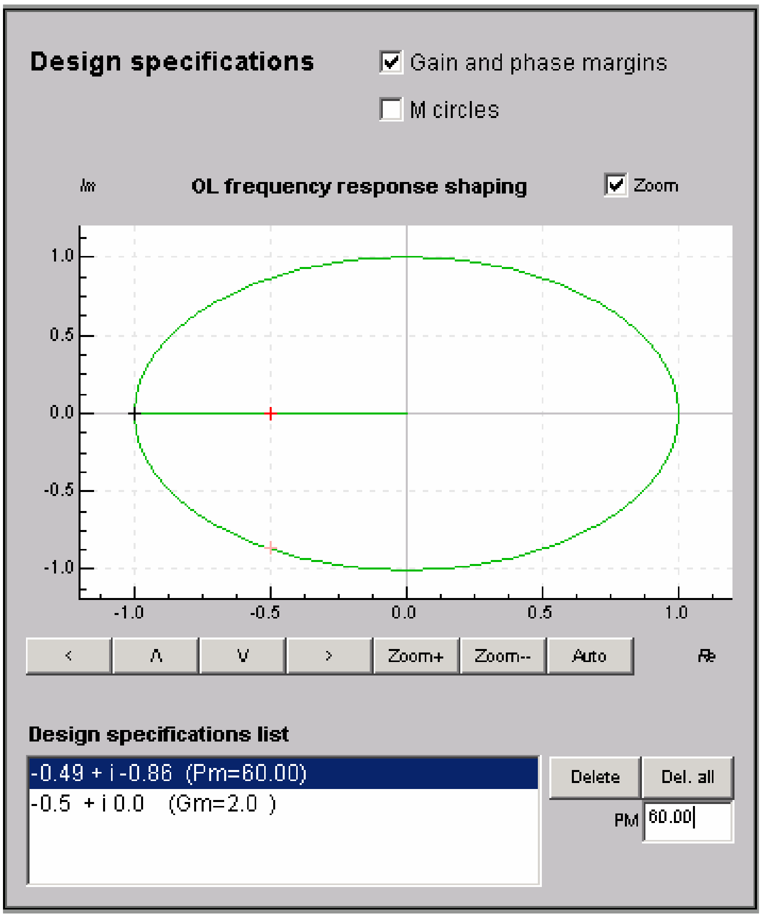
We specify time the constants, gain and transport delay in BODE form. After that, we need to confirm new model by the OK button.



2) Design specifications

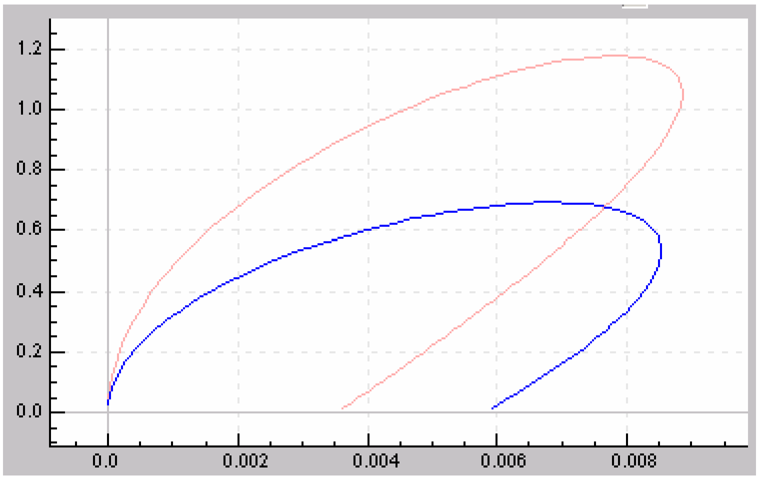
We use default values of controller type (PID), N and f. Then we specify gain and phase margins (P_m=60, G_m=2) by mouse clicking in DS window. We can set the P_m and G_m values more precisely manually after selection in design specifications list.

If we want to see sensitivity and complementary sensitivity functions restrictions (so called m-circles), we must check the m-circles checkbox. The required Nyquist plot shaping can be reached by choosing points on these circles.



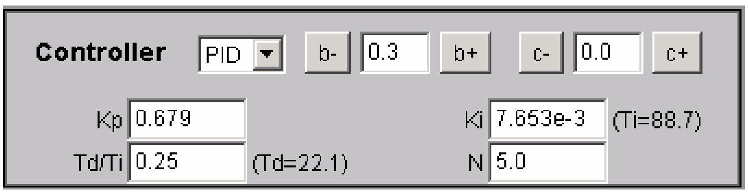
3) Choice of PID controller parameters K_p, K_i in parameter plane

We will see two regions corresponding to the gain and phase margins shaping points. The region actually selected in design specification list is painted by red color. If we want to fulfill both conditions, we must choose the parameters at the regions intersect by mouse clicking.



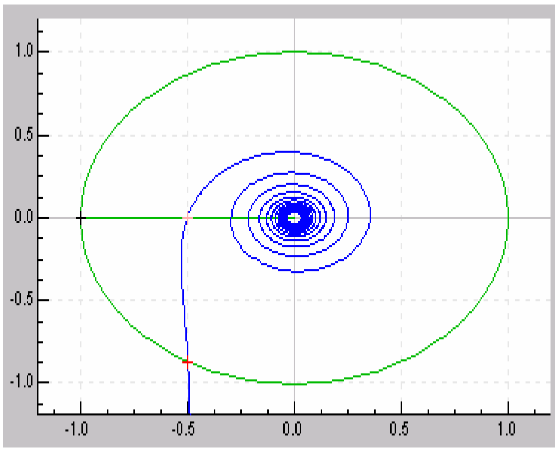
4) Manual tuning of PID parameters

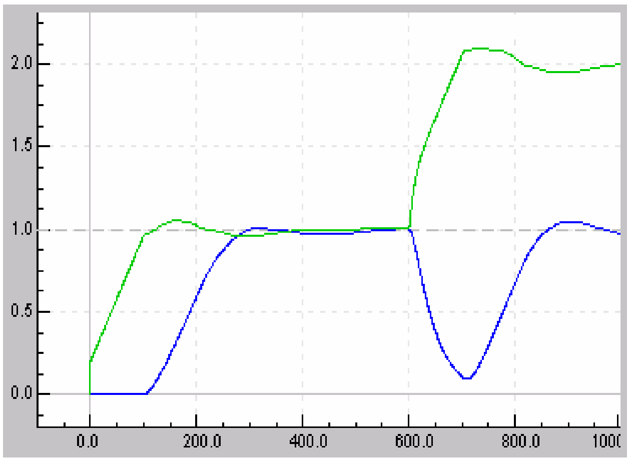
We will see all PID parameters after clicking in RP window. Parameters b and c are set at default values b = 1, c = 0. If we set the parameter b = 0.3, the closed loop behavior will be without overshot. We see the closed loop



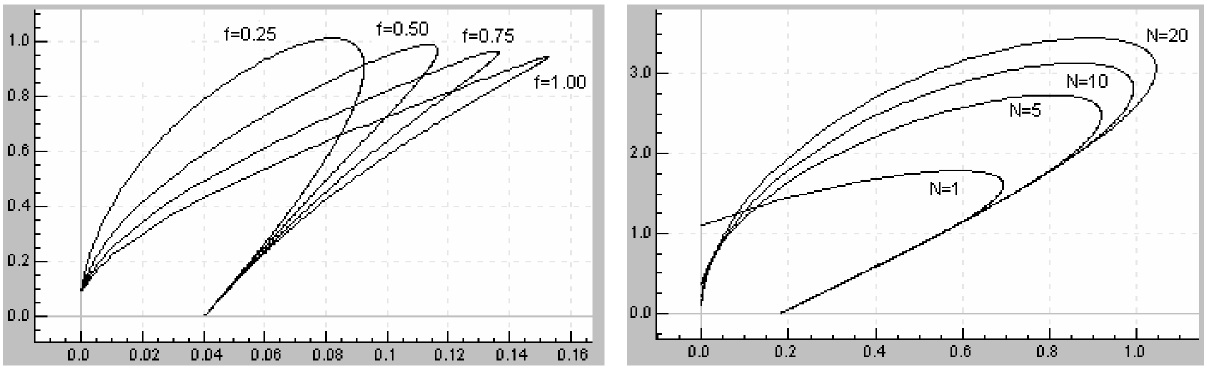
response in LP window. The process variable is painted blue and the manipulating variable is painted green.

Finally, we can ensure that the Nyquist plot passes through both points and the closed loop has required performance.





Let us look now, how the region shape depends on values *N* and *f*. Note that the shape is strongly influenced (mainly for processes without dead time) by the value of derivative part filter parameter *N*. It is clear, that most of methods based on ideal PID controller will not work properly, because the influence of *N* can not be ignored.



(a) (b)

Fig. 4 Dependency of the shape of the region on values *N* and f for process $1/(s+1)^3$ with time delay 10 (4a) resp. 0 (4b) and shaping point X_2 ($Pm = 60^\circ$).

5. Conclusion

The main purpose of this paper is to introduce a new tool for the PID controller design (free accessible Java applet on www.pidlab.com) based on new Nyquist plot shaping method. This method allows to design the real 2DOF PID controller for practical requirements, e. g. gain and phase margins. But the method allows to specify more complicated Nyquist plot shape requirements. The method is usable for any linear system (unstable, non-minimum phase, with or without dead time). This method is useful especially for stable non-oscillatory or slightly oscillatory processes, where the Nyquist plot shape requirements are well known.

References

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